International Journal of Advanced Research in Arts, Science, Engineering & Management (IJARASEM)

| ISSN: 2395-7852 | www.ijarasem.com | Bimonthly, Peer Reviewed & Referred Journal|

| Volume 6, Issue 6, November 2019 |

Prediction of Reliability Index and Probability of Failure for Reinforced Concrete Beam Subjected To Flexure

Salma Taj[1], Karthik B.M.[1], Mamatha N.R.[1], Rakesh J.[1]

[1]UG Students, Dept. of Civil Engineering, Rajeev Institute of Technology, Hassan, Karnataka, India

ABSTRACT: The present method of designing rectangular reinforced concrete beam is based on limit state design philosophy which makes use of partial safety factors for material strength and load. The design variables being random, it becomes much more important to assess the level of safety in the probabilistic design situation. Beam being the vital most structural element, the probability of failure of a beam is linked to the overall safety of a structural system. With this in view, an attempt is made to assess the safety levels in terms of reliability index and probability of failure of the beam. A reinforced concrete frame is modelled in ETABS software, the moment and area of tension reinforcement values are extracted from the same software for statistical analysis. Resistance statistics for rectangular reinforced concrete beam are generated using the equations provided in IS 456-2000 code. The variables relating to geometry, material properties and loading are considered as random. Probability of failure is obtained by Monte Carlo Simulation technique which establishes the statistics of safety margin that is Resistance(R) > Action (S). The study investigates the reliability index and probability of failure of the rectangular reinforced concrete beam and plotting of the histogram and probability distribution curve. The entire reliability analysis was implemented through developing a program in MATLAB software.

KEYWORDS: Reliability Analysis, Probability of Failure, Monte-Carlo Stimulation.

I. INTRODUCTION

A R.C. beam is an important component of reinforced concrete structures. In general, beam may be defined as a flexural member, which resists loads mainly by bending. The evaluation of safety of a beam is much important task. The safety of a beam depends on the resistance 'R', of the beam and action 'S' (load or load effects) on the beam. The action is a function of loads (live load, dead load and super dead.) which are random variables. R.C.Beam is a structural element that primarily resists loads applied laterally to the beam axis. Its mode of deflection is primarily by bending.

Similarly, the resistance or response of the R.C. beam depends on the physical properties of the materials, and geometrical dimensions of R.C. beam which are also subjected to statistical variations, and are probabilistic. Hence to be rational in the estimation of the structural safety, the random variations of the basic parameters are to be taken into account. Since load and strength are random variables, the safety of the R.C. beam is also a statistical variable. In the present work an attempt is made to assess the safety of a R.C. beam by establishing reliability index using Monte Carlo simulation.

Reliability analysis is defined as the consistent evaluation of design risk using probability theory. The reliability is the probability of an item performing its intended function over a given period of time under the operating conditions encountered. It is important to note that the above definition stresses four significant elements namely,

1. Probability
2. Intended function
3. Time
4. Operating conditions

Because of the uncertainties, the reliability is a probability, which is the first element in the definition. The second point, intended function, signifies that the reliability is a performance characteristic. For a structure to reliable, it must perform certain function or functions satisfactorily for which it has been designed. Reliability is always related to time. In case of structure, it must perform the assigned function satisfactorily. The last point is operating condition; this establishes the actions, stresses that will be imposed on the structure. These may be loads, temperature, corrosive atmosphere, shock, etc.

II. METHODOLOGY

A.BASIC ASSUMPTIONS

Design for the limit state of collapse in flexure shall be based on the assumptions given below:

1. The plane sections normal to the axis remain plane after bending. This means that the strain at any point is proportional to its distance from the neutral axis.
2. The relationship between compressive stress distribution in concrete and strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of the strength in substantial agreement with the results of tests. For design purposes, the compressive strength of concrete in structure is assumed to be 0.67 times the characteristic strength. The partial safety factor 1.5 is applied in addition to this. Thus the design strength of concrete is taken as 0.446fc.
3. The tensile strength of the concrete is ignored.
4. The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used. For design purposes, partial safety factor 1.15 is applied.
5. The maximum strain in the reinforcement in the section at failure shall not be less than $\frac{fy}{1.15\text{Es}} + 0.002$.
6. The maximum strain in concrete at the outermost compression fibre is taken as 0.0035 in bending when neutral axis lies within the section.

III. PROBABILITY OF FAILURE OF A STRUCTURAL MEMBER

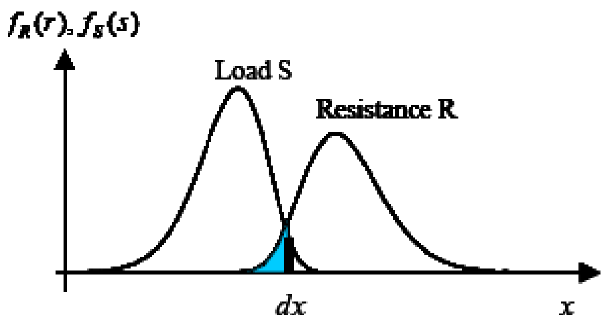


Figure 1: Illustrating overlap of action and resistance distributions indicating failure probability

Consider the fig.1 where probability density functions of Resistance R and Action S are plotted. When R and S are plotted it is seen that both distributions overlap. The shaded portion (overlap) in figure gives an indicative measure of the probability of failure of the element or structure.

IV. MONTE CARLO SIMULATION METHOD

Simulation is the process of replicating the real world problem based on the set of assumption and conceived models of reality. It may be performed theoretically or experimentally. The simulation process yields a special measure of performance or response. Through repeated simulations, the sensitivity of the system performance to variation in the system parameters may be examined. By this procedure, simulation may be used to determine optimal designs. One of the usual objectives in using the Monte Carlo technique is to estimate certain parameters and probability distributions of random variables whose values depend on the Interactions with random variables whose probability distributions are specified. For engineering purposes simulation may be applied to predict or study the performance and for response of a system. With a prescribed set of values for the system parameters or design variables

Monte Carlo simulation is a repeated process of generating deterministic solutions to a given problem; each solution corresponds to a set of deterministic values of the underlying random variables. The main element of a Monte Carlo simulation procedure is the generation of random numbers from a specified distribution. 500 numbers of data sets was randomly generated for each cross section, and each data set varied randomly as a function of statistical models for the variables involved.

A.NORMAL DISTRIBUTION

For Normal distribution, the Box and Muler technique is used to generate normal variates. Here, standard normal deviates are obtained by generating two uniform random numbers V1 and V2 (with a uniform density range between 0 and 1) at a time.

Then the standard normal variates are given below:

$$u1 = [2 \ln 1/v1]^{1/2} \cos (2\pi V_2)$$

$$u2 = [2 \ln 1/v1]^{1/2} \sin (2\pi V_2)$$

Generation of Normal variates from the distribution of Y following the Normal Distribution with mean μ and variance σ.

First generate two uniform random numbers V1 and V2 in the range of 0 and 1. Then the standard normal variates are given by above equations we know that the standard normal variate is connected to the normal variate Y as follows:

$$Y-\mu/\sigma = U$$

Where, U is the Standard normal variate. Hence we can get two normal variates y_1 and y_2, using above equations. Thus,

$$y_1 = \sigma_{u1} + \mu$$

$$y_2 = \sigma_{u2} + \mu$$

That is,

$$y1 = \mu + \sigma\ [2 \ln 1/v_1]^{1/2} \cos (2\pi V_2)$$

$$y2 = \mu + \sigma\ [2 \ln 1/v_1]^{1/2} \sin (2\pi V_2)$$

The method of Monte Carlo simulation is used when dealing with random variables. The procedure is usually repeated to generate a different set of values of the variables in accordance with a specified probability distribution.

SL.NO	VARIABLES	C.O.V.	REFERENCES
1	Geometric variables a) b, mm	0.03	Ravindra P. Patil and K. Manjunath,(2010)"An Empirical Expression for Reliability Index of Flanged RC Beams in Limit State of Deflection"
	b) d, mm c) A_{st}, mm^2	0.05 0.04	Dr. K. Manjunath and Meghana Bharadwaj,(2016) "Reliability analysis of Flanged RC Beams in Limit State of Cracking"
2	Material properties		Dr. K. Manjunath and Meghana Bharadwaj,(2016) "Reliability analysis of Flanged RC Beams in Limit State of Cracking"
	a) f_{ck}, N/mm^2	0.15	
	b) f_y, N/mm^2	0.1	
3	Load		Mohammad Masoud Azhdari Moghaddam,(2012) **"**Analysis of Beam Failure Based on Reliability System Theory Using Monte Carlo Simulation Method**".**
	a) W, kN/m^2	0.22	

Table 1: Statistics of Basic Variables

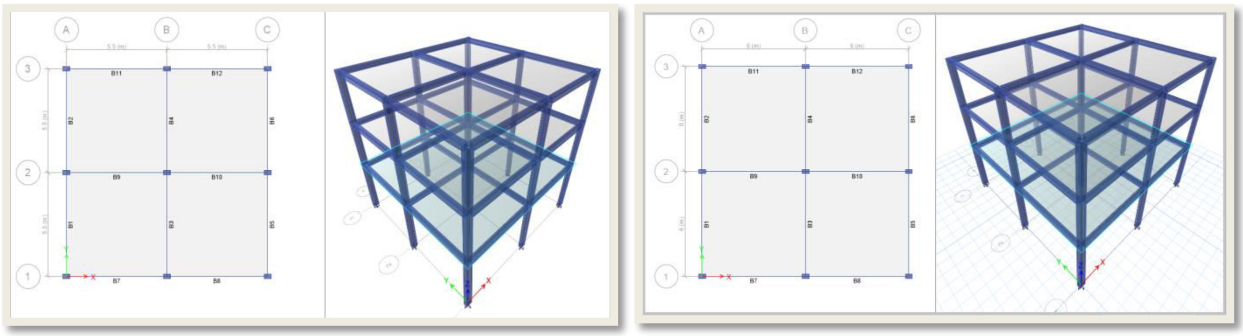
V. GENERATION OF LOAD STATISTICS AND RESISTANCE STATISTICS

In the process of codal assessment, reliability analysis of existing design of beams as per the current codal provisions are carried out for limit state of collapse in flexure and then the reliability levels of the present designs under different design situations are established. Beam subjected to bending is considered. A typical model of a multi- storey building is developed using ETABS.

International Journal of Advanced Research in Arts, Science, Engineering & Management (IJARASEM)

| ISSN: 2395-7852 | www.ijarasem.com | Bimonthly, Peer Reviewed & Referred Journal|

| Volume 6, Issue 6, November 2019 |



(a) (b)

Figure 2(a, b): Plan view of Multistorey building (a) for (11x11) m span (b) for (12x12) m span

Load is applied on the beam in the form of live load and this load is transferred to columns in terms of reactions. Modelling of the resistance variable of a structural element and a structure is a difficult task. The resistance is a function of these basic variables namely geometric dimensions, physical properties etc.

From the software, bending moment values and area of steel reinforcement values are taken for statistical analysis and the same is shown in the below table.

Sl. No.	f_{ck} N/mm^2	Depth (D) mm	Bending Moment kN-m	A_{st} mm^2
1	20	320	46.5942	2869
2	20	330	49.1672	2901
3	20	350	53.6769	2848
4	20	400	64.6205	2777
5	25	320	46.5942	2809
6	25	330	49.1672	2805
7	25	350	53.6769	2792
8	25	400	64.6205	2088

(a)

Sl. No.	f_{ck} N/mm^2	Depth (D) mm	Bending Moment kN-m	A_{st} mm^2
1	20	320	38.4780	2221
2	20	330	40.4401	2214
3	20	350	44.2841	2197
4	20	400	53.3129	2157
5	25	320	38.4780	2173
6	25	330	40.4401	2169
7	25	350	44.2841	2138
8	25	400	53.3129	2097

(b)

Table 2 (a, b): Bending moment and area of tension reinforcement values for varying characteristic strength of concrete and depth of beam. (a)For (12x12) m Span (b) For (11x11) m Span

VI. COMPUTATION OF PROBABILITY OF FAILURE

Histogram provides an immediate impression of the range of the data, its most frequently occurring values and the degree to which it is scattered. It is the presentation of data in useful form. The observations are made and noted down as they occur and hence the collected data will be in an unorganized form. This unorganized data is arranged properly. The values are marked in an increasing order. These ordered values are then divided into intervals and the number of observations in each interval is plotted as a bar.

VII. METHODOLOGY OF FINDING PROBABILITY OF FAILURE OF BEAM

- For random variations in different grades of concrete, dimensions and live load, corresponding bending, moment values are obtained from ETABS software.
- Simulate resistance using equation for beam subjected to transverse load specified in IS 456-2000, i.e., flexure resistance equations:

  For under reinforced section : $M_u=0.87f_{ck}\ A_{st}\ d\ [1-(A_{st}\ f_y/f_{ck}\ bd)]$

  For over reinforced section : For Fe250, $X_{umax}=0.53d$ $M_u=0.149\ f_{ck}bd^2$

  For Fe415, $X_{umax}=0.48d$, $M_u=0.138f_{ck}bd^2$

  For Fe500, $X_{umax}=0.46d$, $M_u=0.133f_{ck}bd^2$
- Compare Resistance (R) and Load (S).

| ISSN: 2395-7852 | www.ijarasem.com | Bimonthly, Peer Reviewed & Referred Journal|

| Volume 6, Issue 6, November 2019 |

- If R < S then it will be considered as failure.
- Compute probability of failure by equation $P.F. = \varphi\left[\frac{\mu_Q - \mu_R}{\sqrt{\sigma_Q^2 + \sigma_S^2}}\right]$
- Reliability index (β) = $\beta = \Phi^{-1}(P_f)$.

SL. NO	Fck N/mm^2	Depth (D) in mm	Reliability index (β)	Probability of failure (P.F.)
1	20	320	3.0895	1.0025×10^{-3}
2	20	330	4.0591	2.4644×10^{-5}
3	20	350	5.7057	5.8101×10^{-9}
4	20	400	9.4416	1.8666×10^{-21}
5	25	320	8.5315	7.3137×10^{-18}
6	25	330	9.2697	9.483×10^{-21}
7	25	350	10.919	4.7918×10^{-28}
8	25	400	14.5874	1.7682×10^{-48}

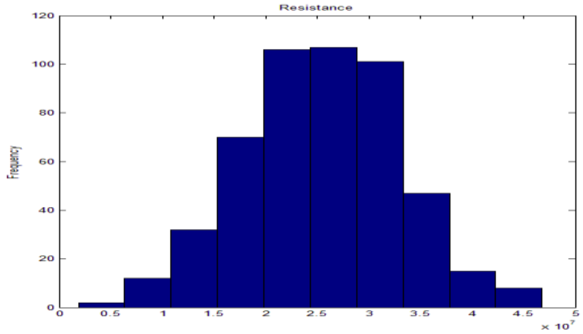
(a)

SL. NO	Fck N/mm^2	Depth (D) in mm	Reliability index (β)	Probability of failure (P.F.)
1	20	320	8.8312	5.25×10^{-19}
2	20	330	9.0977	4.6833×10^{-20}
3	20	350	11.0993	6.4845×10^{-29}
4	20	400	12.3646	2.0985×10^{-35}
5	25	320	12.5017	3.7778×10^{-36}
6	25	330	12.4955	4.0841×10^{-36}
7	25	350	13.5858	2.5297×10^{-42}
8	25	400	14.0142	6.6574×10^{-45}

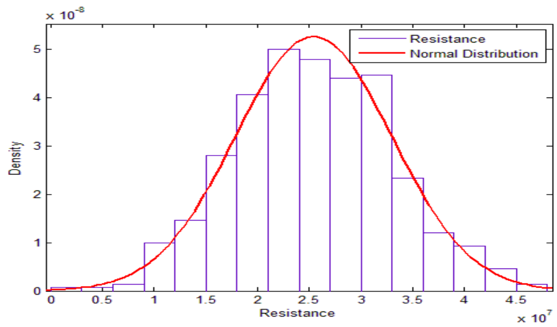
(b)

Table 3(a, b): Reliability index and probability of failure (a) for (12x12) m span (b) for (11x11) m span

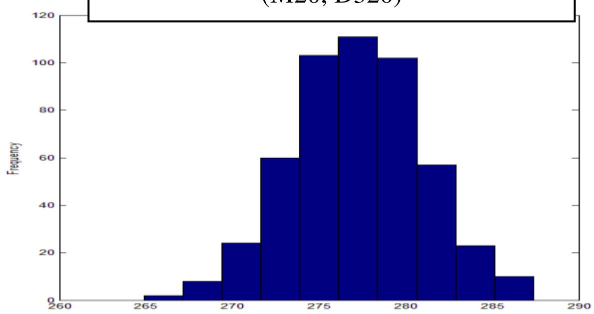
HISTOGRAM AND PROBABILITY DISTRIBUTION



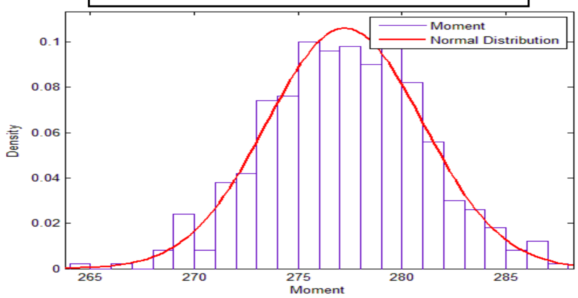
Histogram for beam against resistance (M20, D320)



Probability distribution curve for beam against resistance

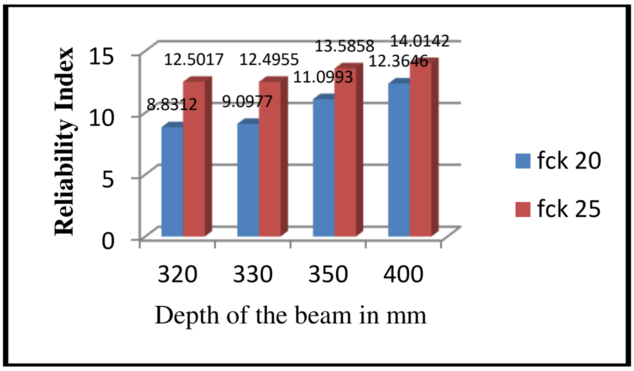


Histogram for beam against moment (M20, D320)

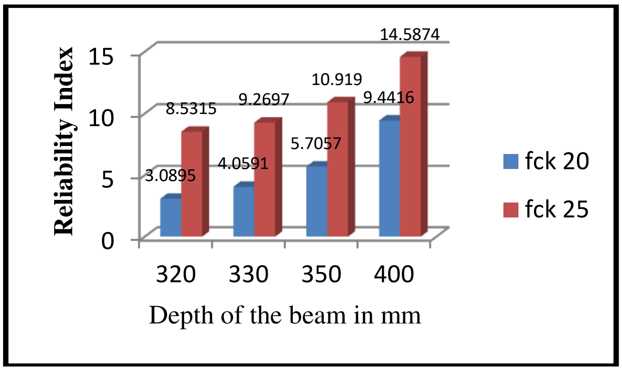


Probability distribution curve for beam against moment

VIII. RESULTS AND DISCUSSION



(a)



(b)

Graph 1(a, b): Variation of Reliability index across depth of the beam for f_{ck} 20 N/mm^2 & 25 N/mm^2 (a) for meshing size of (11x11) m (b) for meshing size of (12x12) m

IX. CONCLUSION

Explicit evaluation of safety in terms of probability of failure of a rectangular reinforced concrete beam when the design variables are random in nature and do follow given probability distributions is done using digital simulation by Monte Carlo simulation technique.

1. It is observed that the reliability index varies from 8.8312 to 14.0142 for meshing (span) size of (11 x11) m in flexure which corresponds to a probability of failure of 5.25 x 10^{-19} to 6.6574 x10^{-45} for characteristic strength of concrete of 20 N/mm^2 & 25N/mm^2 with varying depth of beam. It can be seen that as the depth of beam increases irrespective of fck, there is an increment in reliability index which leads to lesser value in probability of failure.

2. It is observed that the reliability index varies from 3.0895 to 14.5874 for meshing (span) size of (12 x12) m in flexure which corresponds to a probability of failure of 1.0025 x 10^{-3} to 1.7682 x10^{-48} for characteristic strength of concrete of 20 N/mm^2 & 25 N/mm^2 with varying depth of beam. It can be seen that as the depth of beam increases irrespective of fck, there is an increment in reliability index which leads to lesser value in probability of failure.
3. Failure of the beam for both size of meshing is least, when characteristic strength of concrete and depth of the beam are increased.
4. Thus there is almost a consistent level of reliability in the design methodology adopted by IS 456-2000. However the present limit state method of design does not consider the importance of a structure, exposure conditions, effect of quality control etc.

REFERENCES

[1] Dr. K. Manjunath and Meghana Bharadwaj, "Reliability analysis of Flanged RC Beams in Limit State of Cracking".
[2] Andre J. Torii and Roberto D. Machado, "Reliability analysis of Nonlinear Reinforced Concrete Beams".
[3] Mohammad Masoud Azhdari Moghaddam, "Analysis of Beam Failure Based on Reliability System Theory Using Monte Carlo Simulation Method".
[4] Hassan Baji and Hameed Reza Ronagh, "Reliability Analysis of Moment Redistribution in RC Beams".
[5] Renjian Lu.et.al, "Reliability evaluation of Reinforced concrete beams".
[6] M. R. Machado and J. M. C. Dos Santos, "Reliability Analysis of Damaged Beam Spectral Element with Parameter Uncertainties".
[7] Ravindra P. Patil and K. Manjunath, "An Empirical Expression for Reliability Index of Flanged RC Beams in Limit

State of Deflection".
[8] Dr. B. C. Punmia, Ashok Kumar Jain and Arun Kumar jinn – Limit State Design of Reinforced Concrete – First edition, Laxmi Publications (p) limited.
[9] Limit state Design of Reinforced Concrete, 2nd edition, by P.C.Varghese, Prentice- Hall of India Pvt.Ltd. New Delhi, 2002.
[10] IS 456:2000 "Indian Standard Code of Practice for Plain and Reinforced Concrete" (Fourth revision), Bureau of Indian Standards.