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Mathematical Operators

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ABSTRACT: Math operators define the basic operations that act on numbers and other math constructs. Typically, operators take one or two expressions as input and calculate a result as output. For two mathematicians to produce the result when given the same expression, the order of operations is defined so that the result is unambiguous.

KEYWORDS: Mathematical, operators, calculate, expression, order, unambiguous

I. INTRODUCTION

The basic arithmetic operators are addition, subtraction, multiplication, and division. Introduced in elementary mathematics, they describe ways to manipulate numbers. As the notion of a number becomes more complex, their definitions are expanded beyond discrete numbers and properties are introduced to make the concept of a number and operations more cohesive. For example, in algebra, the operators are expanded to work with fractions and partial numbers.[1,2,3]

Addition Operator
Addition is a basic operation in mathematics for combining two numbers together. It is a binary operation denoted with the plus symbol with an expression on the left and an expression on the right.

Subtraction Operator
Subtraction is a basic arithmetic operation of taking away one number from another number.

Multiplication Operator
Multiplication is a basic arithmetic operation performed on two numbers. Multiplying a number by another number is the same as taking n groups of the other number.

Division Operator
The division operator returns the result of dividing one number by another number.

Algebra
These operators in the algebra section build upon the previous operators in the arithmetic section and add a variety of operators used to express more complicated patterns that appear in mathematics. For example, the factorial operator represents the patterns found in combinations and permutations. Another example is the exponent and logarithm operators which describe exponential growth and decay.

Absolute Value Operator
The absolute value operator returns the distance from zero on the number line of a number.

Square Root Operator
Returns the square root of the provided expression.

Radical Operator
The radical operator returns the n-th root of the provided expression. The radical operator is an alternative way of writing a fractional exponent.

Exponent Operator
The exponentiation operator is a binary operator. The base is an expression or number that is being raised to some exponent. The exponent expression is denoted using superscript text.

Logarithm Operator
Taking the logarithm of a number is the inverse operation of exponentiation. The subscript number is the base of the logarithm and the expression is the operand.

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Factorial Operator
The factorial operator is represented using the exclamation mark. The operator is unary, meaning that it only operates on one expression. The operator is useful when calculating combinations and permutations.
Modulus Operator
The modulus operator returns the remainder of dividing the first expression by the second expression.[4,5,6]

Summation Operator
The summation operator is represented by the summation symbol $\sum$ and represents the operation of summing a sequence of expressions together.

Product Operator
The product operator is represented by the $\prod$ (product) symbol and is used to represent the operation of multiplying a sequence of expressions together.
Linear Algebra
Linear algebra is a branch of mathematics that deals with vectors, vector spaces, linear transformations, and systems of linear equations. These operators act on vectors and matrices to produce results like transformations.

Vector Addition Operator
Vector addition is the operation of adding two vectors together. Visually, the operation can be thought of as placing the two vectors "tip to tail" and drawing the resulting vector.

Cross Product Operator
The cross product operates on two vectors and produces another vector as a result.

Magnitude Operator
The magnitude (or length) of a vector gives a scalar representation of its size, irrespective of its direction.

Dot Product Operator
The dot product, also known as the scalar product, takes two vectors and returns a scalar. It measures the extent to which one vector goes in the direction of another.

Matrix Multiplication Operator
Matrix multiplication is a way to compose linear transformations. In order to multiply two matrices, the number of columns in the first matrix must be equal to the number of rows in the second matrix.

Matrix Transpose Operator
The transpose of a matrix is an operator which reflects a matrix across its diagonal.

Matrix Determinant Operator
The determinant operator calculates a scalar value from a square matrix.
Calculus
These calculus operators define the basic operations such as finding the rate of change of a function or the area underneath a curve.

Limit Operator
The limit operator describes the result of an expression as a variable approaches a value. The operator is used in calculus to formalize what mathematicians mean by approach.

Integral Operator
An integral can be geometrically interpreted as the area under the curve of a function between the two points a and b. Integrals are a core operator in calculus and are used throughout physics and higher-level mathematics.

Derivative Operator
The derivative is one of the main operators in calculus.
Boolean Logic
The boolean logic operators operate on boolean expressions - values that are either true or false. Typically, the binary boolean operators take in two boolean values and return a boolean value as a result. In computing, numbers and more complex data can be compared with data of the same type to test for lexicographical order (less than, greater than) and equality to produce a boolean value.

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Logical And Operator
The logical and operator returns true if both the left side expression and the left side expression evaluate to true, otherwise the operator returns false.

Logical Or Operator
The logical "or" operator returns true if either the left-side expression evaluates to true or the right-side expression evaluates to true, otherwise returns false.[7,8,9]

Logical Exclusive Or Operator
The logical exclusive or (abbreviated as xor) operator returns true if the left side evaluates to true and the right side evaluates to false. The operator also returns true if the left side evaluates to false and the right side evaluates to true. Otherwise, returns false.

Logical Implies Operator
The logical implication operator returns true if the left and right-hand side expressions evaluate to true, or if the left-hand expression is false.

Logical If and only If Operator
The logical if and only if operator, or iff for short, returns true if both A and B are false or if both A and B are true.
Set
The set operators are binary operators used in set theory to operate on sets.

Union Operator
The set union operation is denoted using the cup symbol. The union of two sets returns the combined elements of both sets. Duplicates are ignored.

Intersection Operator
The set intersection operator returns the shared elements between two sets. The operator is denoted using the cap symbol.

II. DISCUSSION

A mathematical operator is a symbol that stands for carrying out one or more mathematical operations on some function. For example, we can use the symbol d/dx to stand for the operation of differentiating with respect to x. When an operator operates on a function, the result is generally another function. We will usually assign a symbol to an operator that consists of a letter with a caret (ˆ) over it. We will discuss three types of operators:
1.Multiplication operators are operatorsthat stand for multiplying a function either by a constant or by another function.
2.Derivative operators stand for differentiating a function one or more times with respect to one or more independent variables.
3.Symmetry operatorsare defined by the way they move a point in space but can also operate on functions.

Example 13.1
For the operator
A^=x+dd$x$
find A^f if f=$a$sin(bx), where a and b are constants.
A^$a$sin(bx)=x+ddxasin(bx)=xasin(bx)+abcos(bx).[10,11,12]
13.1.1 Eigenfunctions and Eigenvalues
If the result of operating on a function with an operator is a function that is proportional to the original function, the function is called an eigenfunction of that operator, and the proportionality constant is called an eigenvalue.[1] If A^ is an operator that can operate on a function f such that

$$\hat{A} f = af \quad (13.1)$$

then f is an eigenfunction of A^ and a is the eigenvalue corresponding to that eigenfunction. An equation like Eq. (13.1) is called an eigenvalue equation. The time-independent Schrödinger equation of quantum mechanics is an eigenvalue equation, and other eigenvalue equations are also important in quantum mechanics.
Example 13.2
Find the eigenfunctions and eigenvalues for the operator d2/dx2.
We need to find a function (x) and a constant b such that

d2fdx2=bf.
A trial solution that will work is
f=$e\lambda t$.
Substitution of this trial solution into the differential equation gives
d2$e\lambda t$dx2=λ2$e\lambda t$=$be\lambda t$.
The characteristic equation is
λ2=b,=±b.
The general solution to the differential equation is
(x)=Aexpbx+Bexp-bx,
where A and B are constants. Each of the two terms is an eigenfunction, so we write
f1(x)=Aexpbx,f2(x)=Bexp-bx.
The two eigenfunctions correspond to the same eigenvalue, equal to b. Since no boundary conditions were stated, the eigenvalue can take on any value, as can the constants A and B.
Exercise 13.1
Find the eigenfunctions and eigenvalues of the operator $i$dd$x$, where i=-1.
13.1.2 Operator Algebra
Although a mathematical operator is a symbol that stands for the carrying out of an operation, an operator algebraexists in whichwe manipulate the operator symbols much as we manipulate numbers and symbols in ordinary algebra. We will sometimes write operator equations in which the operators occur without explicit mention of the functions on which they operate. We define the sum of two operators:If we write the operator equation
(13.2)C^=A^+B^
then
(13.3)C^f=(A^+B^)f=A^f+B^f,
where A^ and B^ are two operators and where f is a function on which A^ and B^ can operate.
The product of two operatorsis defined as the successive operation of the operators, with the one on the right operating first. If we write the operator equation
(13.4)C^=A^B^
then
(13.5)C^f=A^(B^f).
The result of B^ operating on f is in turn operated on by A^ and the result is said to equal the result of operating on f with the product A^B^.
Example 13.3
Find the operator equal to the operator product ddxxˆ.
We take an arbitrary differentiable function f=(x) and apply the operator product to it,
ddxxˆf=xdfdx+fdxdx=$x$dd$x$+E^f,
where E^ is the identity operator, defined to be the operator for multiplication by unity (same as doing nothing). The symbol E^ is chosen from the German word "Einheit," meaning "unity."The operator equation that is equivalent to this equation is
ddxxˆ=$x$dd$x$+E^.
As in this example we will usually omit the caret symbol over a multiplication operator.
Exercise 13.2
Find the operator equal to the operator product d2dx2x.
The difference of two operators is given by
(13.6)(A^-B^)=A^+(-E^)B^.
Operator multiplication is associative. This means that if A^,^, and C^ are operators, then
(13.7) $\boxed{(\widehat{A}\,\widehat{B})\widehat{C} = \widehat{A}(\widehat{B}\widehat{C})}$.
Operator multiplication and addition are distributive. This means that if A^,^, and C^ are operators.
(13.8) $\boxed{\widehat{A}(\widehat{B}+\widehat{C}) = \widehat{A}\widehat{B}+\widehat{A}\widehat{C}}$.
Operator multiplication is not necessarily commutative. This means that in some cases the same result is not obtained if the sequence of operation of two operators is reversed:
(13.9) $\boxed{\widehat{A}\widehat{B} \neq \widehat{B}\widehat{A} \quad \text{(possible)}}$.
If the operator A^B^ is equal to the operator B^A^ then A^ and B^ are said to commute. The commutator of A^ and B^ is denoted by A^,^ and defined by
(13.10) $\boxed{[\widehat{A},\widehat{B}] = \widehat{A}\widehat{B} - \widehat{B}\widehat{A} \quad \text{(definition of the commutator)}}$.

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If A^ and B^ commute, then A^,^=0ˆ, where 0ˆ is the null operator,equivalent to multiplying by zero.
Example 13.4
Find the commutator dd$x$,.
We apply the commutator to an arbitrary differentiable function (x):
(13.11)dd$x$,xf=dd$x$(xf)-xdfdx=xdfdx+f-xdfdx=f.
Therefore,
(13.12)dd$x$,x=E^.
Exercise 13.3
Find the commutator [x2,d2dx2].
Here are a few facts that will generally predict whether two operators will commute:
•An operator containing a multiplication by a function of x and one containing d/dx will generally not commute.
•Two multiplication operators commute. If g and h are functions of the same or different independent variables or are constants, then
(13.13)[gˆ,hˆ]=0.
•Operators acting on different independent variables commute. For example,
(13.14)$x$dd$x$,dd$y$=0.
•An operator for multiplication by a constant commutes with any other operator.
An operator raised to the nth power stands for n successive applications of the operator:
(13.15) $\widehat{A}^n = \widehat{A}\widehat{A}\widehat{A}\cdots\widehat{A} \quad (n \text{ factors})$.
Example 13.5
If A^=x+dd$x$, find A^2.
A^2=x+ddxx+dd$x$,=x2+ddxx+$x$dd$x$+d2dx2.
The order of the factors in each term must be maintained because the two terms in the operator do not commute with each other.
Exercise 13.4
If A^=x+dd$x$, find A^3.
Example 13.6
For the operator A^=x+dd$x$, find $A^{2}f$ if (x)=sin(ax).
x2+ddxx+$x$dd$x$+d2dx2sin(ax)=x2sin(ax)+sin(ax)+xacos(ax)+xacos(ax)-a2sin(ax),=x2sin(ax)+sin(ax)+2xacos(ax)-a2sin(ax).
Exercise 13.5
Find an expression for B^2 if B^=(d/dx) and find $B^{2}f$ if f=bx4.
Division by an operator is not defined. However, we define the inverse of an operator as the operatorthat "undoes" what the first operator does. The inverse of A^ is denoted by A^-1:
(13.16) $\widehat{A}\widehat{A}^{-1} = \widehat{A}^{-1}\widehat{A} = \widehat{E}$.[13,14,15]
Note that A^ is also the inverse of A^-1.
(13.17) $\widehat{A}\widehat{A}^{-1} f = \widehat{A}^{-1}\widehat{A} f = \widehat{E} f = f$.
Not all operators possess inverses. For example, there is no inverse for multiplication by zero.
Operator algebra can be used to solve some differential equations.[2] A linear differential equation with constant coefficients can be written in operator notation and solved by operator algebra.
Example 13.7
Solve the equation
d2ydx2-3dydx+2y=0
using operators.
The equation can be written as
D^x2-3D^x+2y=0,
where the symbol D^x stands for d/dx. The equation can be written as an operator equation:
D^x2-3D^X+2=0.
We factor this equation to obtain
(D^x-2)(D^x-1)=0.
The two roots are obtained from
D^x-2=0,D^x-1=0.
These equations are the same as
dydx-2y=0,dydx-y=0.
The solutions to these equations are
y=e2x,=ex.

Since both of these must be solutions to the original equation, the general solution is
y=c1e2x+c2ex,
where c1 and c2 are arbitrary constants.
Exercise 13.6
Show that the solution in the previous example satisfies the original equation.
13.1.3 Operators in Quantum Mechanics
One of the postulates of quantum-mechanical theory is that for every mechanical variable there is a corresponding mathematical operator. For example, the operator that corresponds to the mechanical energy is the Hamiltonian operator, and the time-independent Schrödinger equation is the eigenvalue equation for this operator. For motion in the x direction of a single particle of mass m with a potential energy given by (x), the Hamiltonian operator is
(13.18)H^=-$\hbar$2$m\partial$2∂x2+(x),
where $\hbar$ stands for Planck's constant divided by 2π. The term (x) stands for multiplication by the potential energy function. The operator for the x coordinate is multiplication by x. The operator for the x component of the linear momentum is
(13.19)p^x=$\hbar i\partial\partial x$
where i is the imaginary unit, equal to -1.
Example 13.8
Find an eigenfunction of p^x.
Denote the eigenfunction by f.
p^xf=$\hbar i\partial f\partial x$=af,$x$=ia$\hbar f$,1$f\partial f\partial x$=ia$\hbar$,ln($f$)=iax$\hbar$+$C$,$f$=$eCe$iax/$\hbar$,
where C is an arbitrary constant. Since no boundary condition was specified, the eigenvalue a can take on any value.
Exercise 13.7
Find the eigenfunction of the Hamiltonian operator for motion in the x direction if (x)=E0=constant.
All of the quantum-mechanical operators are hermitian.[3]If an operator, A^, is hermitian, it obeys the relation
(13.20)∫χ∗A^ψdq=∫A^∗χ∗ψdq,
where q stands for all of the coordinates on which the functions χ and ψ depend. The asterisk (*) stands for taking the complex conjugate. The integrals in this formula must be taken over all of the values of the coordinates, and if the coordinates can become infinite, the integrals must converge.
Example 13.9
Show that the operator d/dx is not hermitian.
We assume that x can range over all real values and integrate by parts:
∫-∞∞χ∗dψdxdx=χ∗ψ-∞∞-∫-∞∞dχ∗d$x\psi$dx.
In order for the integrals to converge, the functions χ and ψ must vanish for infinite values of x, so that
∫-∞∞χ∗dψdxdx=-∫-∞∞dχ∗d$x\psi$dx.
This is the negative of what is required, so that the operator is not hermitian.

III. RESULTS

Numerical Mathematical Operations
In solving a typical physical chemistry problem we are usually required to carry out numerical operations. This elementary discussion of these operations can be skipped without loss of continuity.
1.3.1 Binary Arithmetic Operations
These binary mathematical operations involving pairs of numbers are addition, subtraction, multiplication, and division. Some rules for operating on numbers with sign can be simply stated:
•The sum of two numbers of the same sign is obtained by adding the magnitudes and assigning the sign.
•The difference of two numbers is the same as the sum of the first number and the negative of the second.
•The product of two factors of the same sign is positive, and the product of two factors of different signs is negative.
•The quotient of two factors of the same sign is positive, and the quotient of two factors of different signs is negative.
•Multiplication is commutative, which means that[4]

(1.1) $a \times b = b \times a$

where a and b represent two numbers.
•
Multiplication is associative, which means that

(1.2) $a \times (b \times c) = (a \times b) \times c$

•
Multiplication and addition are distributive, which means that

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(1.3) $a \times (b + c) = a \times b + a \times c$

1.3.2 Additional Numerical Operations[16,17,18]

In addition to the four binary arithmetic operations, there are some important mathematical operations that involve only one number (unary operations). The magnitude, or absolute value, of a scalar quantity is a nonnegative number that gives the size of the number irrespective of its sign. It is denoted by placing vertical bars before and after the symbol for the quantity. This operation means that the magnitude of x is given by

(1.4)|𝑥|={𝑥if 𝑥⩾0−𝑥if 𝑥<0

For example,

|4.5|=4.5|−3|=3

The magnitude of a number is always nonnegative (positive or zero).

An important set of numerical operations involving a single number is the taking of powers and roots. If x represents some number that is multiplied by itself 𝑛−1 times so that there are n factors, we represent this by the symbol 𝑥𝑛, representing x to the nth power. For example,

(1.5)𝑥2=𝑥×𝑥,3=𝑥×𝑥×𝑥,𝑥𝑛=𝑥×𝑥×𝑥×⋯×𝑥(𝑛factors)

The number n in the expression 𝑥𝑛 is called the exponent of x. An exponent that is a negative number indicates the reciprocal of the quantity with a positive exponent:

(1.6) $x^{-1} = \frac{1}{x}, \quad x^{-3} = \frac{1}{x^3}$

There are some important facts about exponents. The first is

(1.7) $x^a x^b = x^{a+b}$

where x, a, and b represent numbers. We call such an equation an identity, which means that it is correct for all values of the variables in the equation. The next identity is

(1.8) $(x^a)^b = x^{ab}$

Roots of real numbers are defined in an inverse way from powers. For example, the square root of x is denoted by 𝑥 and is defined as the number that yields x when squared:

(1.9)(𝑥)2=𝑥

The cube root of x is denoted by 𝑥3 and is defined as the number that when cubed (raised to the third power) yields x:

(1.10)(𝑥3)3=𝑥

Fourth roots, fifth roots, and so on are defined in similar ways. The operation of taking a root is the same as raising a number to a fractional exponent. For example,

(1.11)𝑥3=𝑥1/3

The order of taking a root and raising to a power can be reversed without changing the result

(1.12)(𝑥3)3=(𝑥1/3)3=𝑥=(𝑥3)1/3=𝑥33

We say that these operations commute with each other.

There are two numbers that when squared will yield a given positive real number. For example, 22=4 and (−2)2=4. When the symbol 𝑥 is used, the positive square root is always meant. To specify the negative square root of x, we write −𝑥. If we confine ourselves to real numbers, there is no square root, fourth root, sixth root, and so on, of a negative number. In a later chapter, we discuss imaginary numbers, which are defined be square roots of negative quantities. Both positive and negative numbers can have real cube roots, fifth roots, and so on, since an odd number of negative factors yields a negative product.

IV. CONCLUSION

These binary mathematical operations involving pairs of numbers are addition, subtraction, multiplication, and division. Some rules for operating on numbers with sign can be simply stated:

•The sum of two numbers of the same sign is obtained by adding the magnitudes and assigning the sign.
•The difference of two numbers is the same as the sum of the first number and the negative of the second.
•The product of two factors of the same sign is positive, and the product of two factors of different signs is negative.
•The quotient of two factors of the same sign is positive, and the quotient of two factors of different signs is negative.
•Multiplication is commutative, which means that[4]

(1.1) $a \times b = b \times a$

where a and b represent two numbers.

•

Multiplication is associative, which means that

(1.2) $a \times (b \times c) = (a \times b) \times c$

•

Multiplication and addition are distributive, which means that

(1.3) $a \times (b + c) = a \times b + a \times c$ [19,20]

REFERENCES

1. Rudin, Walter (1976). "Chapter 9: Functions of several variables". Principles of Mathematical Analysis (3rd ed.). McGraw-Hill. p. 207. ISBN 0-07-054235-X. Linear transformations of X into X are often called linear operators on X .
2. ^ Roman, Steven (2008). "Chapter 2: Linear Transformations". Advanced Linear Algebra (3rd ed.). Springer. p. 59. ISBN 978-0-387-72828-5.
3. ^ Schey, H.M. (2005). Div, Grad, Curl, and All That. New York, NY: W.W. Norton. ISBN 0-393-92516-1.
4. Halmos 1970, p. 30; the words map, mapping, transformation, correspondence, and operator are sometimes used synonymously.
5. ^ Halmos 1970
6. ^ "Mapping". Encyclopedia of Mathematics. EMS Press. 2001 [1994].
7. ^ "function | Definition, Types, Examples, & Facts". Encyclopedia Britannica. Retrieved 2019-08-17.
8. ^ Spivak 2008, p. 39.
9. ^ f Kudryavtsev, L.D. (2001) [1994]. "Function". Encyclopedia of Mathematics. EMS Press.
10. ^ b Taalman, Laura; Kohn, Peter (2014). Calculus. New York City: W. H. Freeman and Company. p. 3. ISBN 978-1-4292-4186-1. LCCN 2012947365. OCLC 856545590. OL 27544563M.
11. ^ Trench, William F. (2013) [2003]. Introduction to Real Analysis (2.04th ed.). Pearson Education (originally; self-republished by the author). pp. 30–32. ISBN 0-13-045786-8. LCCN 2002032369. OCLC 953799815. Zbl 1204.00023.
12. ^ Thomson, Brian S.; Bruckner, Judith B.; Bruckner, Andrew M. (2008) [2001]. Elementary Real Analysis (PDF) (2nd ed.). Prentice Hall (originally; 2nd ed. self-republished by the authors). pp. A-4–A-5. ISBN 978-1-4348-4367-8. OCLC 1105855173. OL 31844948M. Zbl 0872.26001.
13. ^ Larson, Ron; Edwards, Bruce H. (2010). Calculus of a Single Variable. Cengage Learning. p. 19. ISBN 978-0-538-73552-0.
14. ^ Weisstein, Eric W. "Map". mathworld.wolfram.com. Retrieved 2019-06-12.
15. ^ Lang, Serge (1987). "III §1. Mappings". Linear Algebra (3rd ed.). Springer. p. 43. ISBN 978-0-387-96412-6. A function is a special type of mapping, namely it is a mapping from a set into the set of numbers, i.e. into, R, or C or into a field K.
16. ^ Apostol, T.M. (1981). Mathematical Analysis (2nd ed.). Addison-Wesley. p. 35. ISBN 978-0-201-00288-1. OCLC 928947543.
17. ^ James, Robert C.; James, Glenn (1992). Mathematics dictionary (5th ed.). Van Nostrand Reinhold. p. 202. ISBN 0-442-00741-8. OCLC 25409557.
18. ^ James & James 1992, p. 48
19. ^ e Gowers, Timothy; Barrow-Green, June; Leader, Imre, eds. (2008). The Princeton Companion to Mathematics. Princeton, New Jersey: Princeton University Press. p. 11. doi:10.1515/9781400830398. ISBN 978-0-691-11880-2. JSTOR j.ctt7sd01. LCCN 2008020450. MR 2467561. OCLC 227205932. OL 19327100M. Zbl 1242.00016.
20. ^ Quantities and Units - Part 2: Mathematical signs and symbols to be used in the natural sciences and technology, p. 15. ISO 80000-2 (ISO/IEC 2009-12-01)