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# The Best Approaches to Learning Math Facts and Developing Number Sense 

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#### Abstract

Students learn math best when they approach the subject as something they enjoy. Speed pressure, timed testing and blind memorization pose high hurdles in the pursuit of math, according to Jo Boaler, professor of mathematics education at Stanford Graduate School of Education and lead author on a new working paper called "Fluency Without Fear.""There is a common and damaging misconception in mathematics - the idea that strong math students are fast math students," said Boaler, also cofounder of YouCubed at Stanford, which aims to inspire and empower math educators by making accessible in the most practical way the latest research on math learning.Fortunately, said Boaler, the new national curriculum standards known as the Common Core Standards for K-12 schools de-emphasize the rote memorization of math facts. Maths facts are fundamental assumptions about math, such as the times tables $(2 \times 2=4)$, for example. Still, the expectation of rote memorization continues in classrooms and households across the United States. While research shows that knowledge of math facts is important, Boaler said the best way for students to know math facts is by using them regularly and developing understanding of numerical relations. Memorization, speed and test pressure can be damaging, she added.


KEYWORDS: students, math, facts, number sense, education, curriculum, numerical, standards

## I.INTRODUCTION

People with "number sense" are those who can use numbers flexibly, she said. For example, when asked to solve the problem of $7 \times 8$, someone with number sense may have memorized 56 , but they would also be able to use a strategy such as working out $10 \times 7$ and subtracting two 7 s (70-14)."They would not have to rely on a distant memory," Boaler wrote in the paper.In fact, in one research project the investigators found that the high-achieving students actually used number sense, rather than rote memory, and the low-achieving students did not ${ }^{1}$. The reason was that the low achievers are often low achievers not because they know less but because they don't use numbers flexibly."They have been set on the wrong path, often from an early age, of trying to memorize methods instead of interacting with numbers flexibly," she wrote. Number sense is the foundation for all higher-level mathematics. Boaler said that some students will be slower when memorizing, but still possess exceptional mathematics potential."Math facts are a very small part of mathematics, but unfortunately students who don't memorize math facts well often come to believe that they can never be successful with math and turn away from the subject," she said.Prior research found that students who memorized more easily were not higher achieving in fact, they did not have what the researchers described as more "math ability" or higher IQ scores. ${ }^{2}$ Using an MRI scanner, the only brain differences the researchers found were in a brain region called the hippocampus, which is the area in the brain responsible for memorizing facts - the working memory section.But according to Boaler, when students are stressed - such as when they are solving math questions under time pressure - the working memory becomes blocked and the students cannot as easily recall the math facts they had previously studied. This particularly occurs among higher achieving students and female students, she said.Some estimates suggest that at least a third of students experience extreme stress or "math anxiety" when they take a timed test, no matter their level of achievement. "When we put students through this anxiety-provoking experience, we lose students from mathematics," she said. ${ }^{3}$

Boaler contrasts the common approach to teaching math with that of teaching English. In English, a student reads and understands novels or poetry, without needing to memorize the meanings of words through testing. They learn words by using them in many different situations - talking, reading and writing."No English student would say or think that learning about English is about the fast memorization and fast recall of words," she added. In the paper, coauthored by Cathy Williams, cofounder of YouCubed, and Amanda Confer, a Stanford graduate student in education, the scholars provide activities for teachers and parents that help students learn math facts at the same time as developing number sense. These include number talks, addition and multiplication activities, and math cards.
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Importantly, Boaler said, these activities include a focus on the visual representation of number facts. When students connect visual and symbolic representations of numbers, they are using different pathways in the brain, which deepens their learning, as shown by recent brain research."Math fluency" is often misinterpreted, with an over-emphasis on speed and memorization, she said. "I work with a lot of mathematicians, and one thing I notice about them is that they are not particularly fast with numbers; in fact some of them are rather slow. This is not a bad thing; they are slow because they think deeply and carefully about mathematics."She quotes the famous French mathematician, Laurent Schwartz. He wrote in his autobiography that he often felt stupid in school, as he was one of the slowest math thinkers in class.Math anxiety and fear play a big role in students dropping out of mathematics, said Boaler."When we emphasize memorization and testing in the name of fluency we are harming children, we are risking the future of our ever-quantitative society and we are threatening the discipline of mathematics," she said. "We have the research knowledge we need to change this and to enable all children to be powerful mathematics learners. Now is the time to use it." ${ }^{4}$

## II.DISCUSSION

Many upper elementary teachers notice a severe lack of number sense in their students, and that makes high levels of math extremely difficult for students. Number sense refers to "a well organized conceptual framework of number information that enables a person to understand numbers and number relationships and to solve mathematical problems that are not bound by traditional algorithms". There are five components that characterize number sense: number meaning, number relationships, number magnitude, operations involving numbers, and referents for numbers and quantities. This post shares some strategies I use to develop number sense in upper elementary students. It's certainly not too late to develop number sense with third, fourth, and fifth graders. Before we get started, I'd love to share a brief introduction to number sense video by Jo Boaler. Hopefully, you'll be able to use some of the ideas and strategies below to develop a number sense with your upper elementary students. More than likely, you're already doing several of these things! ${ }^{5}$

Browse the Number Sense Strategies:

1. Number Talks
2. Math Facts
3. Conceptual Instruction
4. Communication
5. Time

Number Sense Strategy 1: Number Talks
I heard about number talks several years before I ever tried implementing them. I convinced myself that they were a complete waste of my time and just another buzz word in education. However, I saw other math teachers that I greatly respected begin implementing number talks and sharing the success they were seeing. I was in fear of missing out, so I reluctantly gave number talks a try. To be honest, I did not have any positive expectations.I am now happy to share that I was completely wrong! Number talks now have a valuable place in my classroom. My favorite number talks resource is Number Talks by Sherry Parrish. It is a long, content heavy book, but it's not necessary to read cover to cover. I started by selecting the essential portions and teaching strategies as I learned them. One of my favorite things about number talks is that I don't need any special resources or materials. I use the book for my lessons and my dry erase board. That's it! I quickly realized that number talks are an amazing routine that will help improve students' number sense, and this is as important for upper elementary students as it is primary students. I'm sure we've all had students who have relied on rote algorithms to complete math problems, but these students did not understand what they are doing. We've all seen the same error patterns over and over again (forgetting to carry the one, subtracting from bottom to top, etc.) and these errors show how the lack of number sense impacts computation. When we develop number sense, students understand the underlying concepts of math operations. ${ }^{6}$
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When it's time for our number talk, I have students sit on the carpet in front of my whiteboard. My rug isn't quite large enough for my whole class, but we improvise the best we can. When students come to the rug, they don't need to bring anything with them, since everything is done mentally. I use my Number Talks book to select 2-4 math problems for my students to solve for the day. I follow the order of suggested lessons in the book, but I'm sure you can skip around too! Next, I write one of the math problems on the board. I typically write the problem horizontally to discourage students from using the algorithm. As students think about the problem, they hold their fist to their chest, and when they determine the solution, they replace their fist with a thumbs up. While students are waiting for the rest of the class to solve the problem, they should think of other strategies they could use and indicate the number of strategies they've though of by holding up two, three, or even four fingers to their chest. I like this procedure, because I can see who has solved the problem and who is thinking. It also prevents students from becoming intimidated by students who solve the problems quickly.

I ask students to share their answers, and I write all of the answers on the dry erase board. I then have students share how they solved the problem. As students explain their thinking, I write what they are telling me they did. After we go through the problem, students share if the strategy worked or not. I explicitly teach the strategies from the Number Talk book before I expect students to apply that particular strategy, which is completely different from how I typically teach math. In a normal math lesson, I want students to construct their own rules and strategies, but this is a bit different. As I teach each strategy, I add them to our number talk anchor chart. The only bit of an issue that we ran into was that my students were coming up with too many different strategies. Our number talks could have gone on forever if I let them. However, I used this to guide discussions on the importance of selecting the most efficient strategy and how to determine which strategies were efficient. I can say with complete confidence that number talks are playing a significant role in improving my students' number sense. They've also opened my eyes to why some students continue to make the same errors again and again and why some types of problems cause students so much difficulty. Even if you only have time for them two or three days a week, I strongly encourage everyone to at least give it a try! ${ }^{7}$

As an upper elementary teacher I do focus on the memorization of math facts. I've found that higher levels of math are extremely difficult for students who do not know their basic math facts. However, I'm not a proponent of isolated memorization or high pressure timed tests either. Many of us have standards that include 'fluency' as a goal, and this fluency comes about when students develop number sense. But isolated practice and repetition will not develop number sense in our students.Research tells us that the best mathematics classrooms are those in which students learn math facts through engaging activities that focus on mathematical understanding rather than rote memorization. When students focus on memorization, they often memorize facts without number sense. This means they are prone to making errors and are not flexible in their thinking. The best way to develop fluency with numbers is to develop number sense and to work with numbers in different ways, not to blindly memorize without number sense. There are three strategies I use to help students learn their math facts and develop number sense. One strategy is through my Multiplication Fact Booklets. As students work through their booklets, they gain a conceptual understanding of each multiplication fact. They are able to solve the fact with multiple strategies, begin to observe patterns, and develop a mathematical vocabulary. Another strategy I enjoy is incorporating meaningful multiplication games into my instruction. Games are effective and eliminate the high stress that timed tests bring to many students. Math games can be played for homework, morning work, math centers, or early finishers. I've created a set of Weekly Multiplication Games, and we focus on one game a week. ${ }^{8}$

When I first started teaching, I had a firm rule that for me to consider a student fluent with a math fact, s/he must be able to solve the math fact in 3 seconds or less. Over the years I've learned that some students will be slower when memorizing a fact or recalling a fact, but they still have exceptional math potential. I've shifted my mindset to allow for flexibility and reasoning in students' thinking. I use Xtra Math as a multiplication fact practice tool, and I customize the program so my students have six seconds

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to solve a problem, rather than the defaulted three seconds. This gives my students the opportunity to apply multiplication strategies they've learned and builds math confidence with my students. Of course, if a student is sailing through without effort, I can adjust the time to 3 or even 1.5 seconds. I also like to put a spin on multiplication flash cards. With this twist, students match the cards with the same numerical answer. This activity encourages an understanding of multiplication as well as rehearsal of math facts. This is a great way to practice math facts while developing number sense. To help our students continue to develop number sense and make sense of math, our math instruction must be conceptual, rather than a series of procedures. By far, the book with the biggest impact on my math lessons is Teaching Student-Centered Mathematics by John Van de Walle. The first time I read it, Van de Walle stepped all over my toes. I have many notes in the margins from the first time I read the book where I questioned everything I read. I've since reread the book several times, and acknowledge that I had a lot to learn when I first started teaching. I still do! This book challenges me to continually look at my math lessons and to find ways to improve those lessons.

When I started teaching, I often focued on algorithms, tricks, and shortcuts without first ensuring conceptual understanding. Algorithms and shortcuts are not necessarily bad. However, the key to using them correctly, lies in the sequence of events that occur in learning math concepts. Our math instruction must first ensure that students' conceptual understanding is deeply embedded. When students have truly mastered a concept, and they should be able to show all the detailed steps in a process, explain why those steps occur. Once students reach this point, we can expose them to more efficient ways to express or perform those same processes. When we teach algorithms and shortcuts first, they become the means to get to the answer. When we bypass conceptual understanding, it is harder for students to understand more complex topics as they advance in school. Over the years, I've modified my math instruction to allow students to construct their own understanding. For example, when using maipulatives, I no longer structure my lessons so that students are directed in exactly how to use the manipulatives. When I do this, my students are blindly following directions and look like they understand. But a rote procedure with a model is still a rote procedure. When getting answers rather than solving problems becomes the focus, students will gravitate to the easiest method available to get answers. ${ }^{9}$

This does not help a student's number sense, because the student is not reasoning with numbers. We must allow math to be problematic for students. A problem is a task where students have no prescribed or memorized rules or methods, nor is there a perception by students that there is a specific solution method. These problems focus students' attention on ideas and sense making.

An example of a task that would be problematic for students is this divisibility task. In this assignment, students must add a digit to the ones place to create a number that is divisible by three, and students explain how they chose that number. Yes, I could simply have students memorize divisibility rules, but there is no application or problem in that lesson. We could make a cute craft or sing songs, but without authentically understanding the rule, students will not be able to apply what they have learned. Of course, I'm not criticizing crafts and songs! They're a tool, but they are not enough for conceptual learning. My students ask me all the time why they have to write so much in math! Students should be able to think through what methods make sense and discuss the use of different methods. Communication of math ideas help students solidify their understanding of mathematics. Math can be thought of as a language, and communication plays an important role in making mathematics meaningful.I occasionally have students write about math through Math Journal Prompts. I had my prompts for the year bound into books for my students, and they regularly write in their journals. The prompts are not word problems or multi part problems. They're unique in the sense that they're opened ended and encourage metacognition and evaluation. I also try to regularly incorporate writing in my math tasks. I have students explain how they solved the problem using multiple representations. Getting a good response is typically like pulling teeth at the beginning of the school year. But with extensive modeling and practice, students always come around. I was hesitant to add time to this post, because it's something we don't have a lot of control over as teachers. As I write this, I recognize that I'm preaching
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to the choir. I get it. I'm teaching full time, so I'm right there with you. However, it's hard to mention the topics above without at least mentioning time. We all know that there isn't enough time in the day to do all that we need to do, and there isn't enough time in the school year to effectively teach all of our standards. ${ }^{10}$

One thing we have to look at is our daily schedule. I've heard of classrooms with math blocks of 25 and 30 minutes. With that short of time frame, there is no way to adequately teach math standards and build number sense. Many of us have no control over the time allotted for each subject, but if possible, I recommend at least an hour a day for math. With our daily time crunch, it's important to eliminate as much down time and transition time as possible.Every year I feel I have to move so fast through math standards that my students don't have time to truly understand and conceptualize the standards. Conceptual instruction takes longer than procedural instruction, so I'm not able to rapidly move through the standards. This leaves me in a panic after Christmas, because I have little time left to teach so many things. I'm learning to spend the bulk of my time on my essential standards or power standards. Those are the standards that it's important for students to master for success in math in later years. For me, those are my place value, addition, subtraction, multiplication, division, fraction, and decimal standards. I absolutely teach my other standards, but I don't go as deep as I would like because of time constraints. In my experience, math is a subject that is often short changed. Whether it's professional development, early intervention programs, RTI, or after school tutoring, there always seems to be less of an emphasis on math. I teach reading too, so I'm completely on board with supporting students as much as possible with reading, but I don't think math should be neglected either.I'm learning, not there yet, to not waste my energy complaining or worrying about what $I$ can't control. I'll continue to advocate for high quality math instruction, but if I only focus on what's wrong, I won't make positive gains. I tutor one afternoon a week after school, because I know it's a need for my students. A coworker and I have set aside time for biweekly professional development, because we're not satisfied with where we are in our instruction. We want to continually improve. These are things I can do something about, and the rest....well, maybe one day.I've found that the best way to instill number sense is through my math warm-ups. This incorporates each of these strategies, using only about 10 minutes a day-at most! If you'd like to learn more about my warm-ups, be sure to check this out. ${ }^{11}$

## III.RESULTS

When it comes to teaching math basics, a lot of progress has been made in the way we approach key skills like number sense, but there are some older, less productive practices still making the rounds. Too often, young students are introduced to foundational concepts like addition, multiplication, fractions, and subtraction-their first brush with formal mathematics-using tactics that prioritize speed, memorization, automaticity, and, above all, right answers and accuracy. Right answers matter, of course, and speed, recall, and automaticity underlie long-term fluency, but none of it should come at the expense of mathematical reasoning and procedural (as opposed to factual) fluency, according to the researchers Gina Kling, a mathematics teacher educator at Western Michigan University, and Jennifer Bay-Williams, a professor of mathematics education at the University of Louisville. In a 2021 study, they suggest that many common teaching practices send the message that math is essentially about recall and blind adherence to formulas. Instead, it's our responsibility to bring math to life, they say, by designing learning experiences that emphasize "curiosity, flexibility, and wonder" and position mathematics as a powerful, adaptable tool for making sense of the world. "Long-standing methods for teaching basic facts have not been effective for far too many students," explain Kling and Bay-Williams, who point to "quick fix" approaches that actually hinder math learning, sacrificing a young student's long-term development of number sense in favor of short-term tasks that emphasize rote memorization and speed. The tactics can produce anxiety and longterm math avoidance starting as early as first grade, according to a 2013 study.

Here are six unproductive math practices that teachers should avoid.
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1. Ignoring Visualization Strategies

It's a faulty assumption that you can explain a math strategy verbally and students will get it -but educators "cannot simply tell a student to understand," Kling and Bay-Williams write. More important, students need time and experiences to understand and visualize number relationships, not just a good explanation. ${ }^{12}$

Make liberal use of visual strategies, the researchers say. One fun activity is to use Quick Look cards: Briefly show students groups of several dots-or photos of familiar items like a carton of eggs-and then ask them how many they saw and how they saw them. For example, did students see four groups of two or two groups of four? Seeing numbers represented visually, whether with dots or manipulatives, can help students gain a better understanding of how and why math strategies work.

Or you could try Splat!, designed by Steve Wyborney, a K-12 district math coach in Oregon. Using a series of slides, he first asks his students to count the number of dots they see. He then shows the next slide, which suddenly has a large amoeboid blob covering several dots, and asks, "How many dots have been covered by the splat?" Students often come up with their own strategies-whether counting up or down, or using their voice or fingers.

## 2. Teaching Math Facts In Numerical Order

A common approach is to teach addition and multiplication facts in order of addend or factor size -starting with 0 s , and then moving to 1 s , then 2 s , and 3 s , for example, when learning the times tables. But that's a mistake, because students then tend to see mathematical "facts as isolated objects," write Kling and BayWilliams, which can result in a superficial understanding of mathematical operations and ultimately reduce a student's level of achievement.

They point to research showing that starting with foundational sets- $2 \mathrm{~s}, 10 \mathrm{~s}$, and $5 \mathrm{~s}-\mathrm{is}$ not only more familiar to students but also vital because other math facts can be derived from them. For example, once a student learns their 5 s , they can break down challenging problems—such as $8 \times 7$-into $8 \times 5$ plus $8 \times 2$, and also tackle more daunting problems like $56 \times 8$. After mastering the foundational sets, students should move on to squares ( $7 \times 7$ ) "because some of the toughest facts to learn (e.g., $7 \times 8$ and $6 \times 7$ ) are close to squares, and squares are useful for later work in algebra, geometry, and measurement." ${ }^{2}$

## 3. Sticking With One Strategy To Solve Problems

When students first learn subtraction, they're often taught to convert the problem into an addition problem. For example, if they're solving $15-9=$ ?, they'd start with 9 and think about what they should add to reach 15. This is a good approach, but it's just one strategy out of many, and ignoring the others can inhibit a student's ability to develop richer number reasoning skills.

Get students to mix in other strategies like compensation-converting 15-9 into 15-10 and then adding 1 to the answer, for example. Students should also learn how to break apart the minuend (the number to be subtracted from) and subtrahend (the number to be subtracted). For example, in the original $15-9$ problem, you can break the 15 into 10 and 5 , and show students that they can first solve $10-9$, and then add the remainder of 5 to arrive at the final answer of 6 . Once students have a grasp of compensation and breaking down numbers, they can move up the scale to larger numbers, such as $132-99$, which is the same as $132-$ $100=32$, and then adding 1 . Using multiple strategies supports mathematical sense-making that goes far beyond the immediate math problem.

## 4. Focusing Too Much On Fact Mastery And Recall

Pages densely packed with math problems send the message to students that speed and mechanical mastery are more important than developing a robust sense of math fluency. "Very few students want to do 30+
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problems on a page," write Kling and Bay-Williams, and that first bad taste "leads many students to decide they do not like mathematics."Similarly, placing too much emphasis on memorization and calculation tricks—such as teaching young learners how to quickly multiply by 9 using their fingers-doesn't help with other numbers. While it may be a useful starting point, it's better to help students develop fluency by teaching them reasoning strategies. For example, $69+58$ might seem daunting to solve, but teaching students how to "make tens"-by adding 1 to 69 and subtracting 1 from 58 , transforming the problem into $70+57$-helps them learn how to manipulate numbers more fluidly. You can also spice things up with hands-on games and interactive activities: Math games such as bingo addition and connect four reveal how comfortable students are with applying different mathematical strategies-such as breaking numbers down and estimating-and the reasoning behind their answers, which can greatly increase students' taste for math. ${ }^{4}$

## 5. Placing Too Much Emphasis On Speed

Speed contests such as board races and timed online games are fun and can help young learners practice how to add, subtract, and multiply quickly-but consider using them sparingly.

That's because emphasizing speed too early can "drive fluency development in the opposite direction," explain Kling and Bay-Williams, encouraging students to revert to simple strategies that can be executed fast, like counting, instead of practicing more complex, time-consuming strategies that build flexible mathematical reasoning skills.Balance speed with reflection and mix in fun, low-stress games that help students practice their math skills in a variety of ways. For example, a favorite is Tens Go Fish, in which players look for combinations of two cards that add up to 10 , instead of matching pairs (so if a student has a 7 in their hand, they can ask their opponent for a 3 ). ${ }^{6}$
6. Adding Extra Pressure Through Timed Tests

Math fluency can be stymied when students are forced to finish single-answer math tests quickly, like "asking third graders to solve 100 multiplication problems in three minutes," write Kling and BayWilliams. While timed tests can sometimes be used as quick assessments of fluency, they are poor ways to measure flexible thinking-a hallmark of strong mathematical reasoning-and can also cause anxiety, inhibit clear thinking, unfairly penalize methodical thinkers, and reinforce the idea that mathematics is a dreary, unforgiving discipline. Try using a range of assessment strategies, such as think-pair-share, peer interviews, journaling, open-ended questions, and storytelling instead, which don't have the pernicious effects of timed tests while offering a more accurate picture of what students know. Following up a math problem with "How did you figure it out?" places less emphasis on getting the right answer and communicates to students that their thinking matters. ${ }^{8}$

## IV.CONCLUSIONS

The term "number sense" is a relatively new one in mathematics education. It is difficult to define precisely, but broadly speaking, it refers to "a well organised conceptual framework of number information that enables a person to understand numbers and number relationships and to solve mathematical problems that are not bound by traditional algorithms" . The National Council of Teachers,identified five components that characterise number sense: number meaning, number relationships, number magnitude, operations involving numbers and referents for numbers and quantities. These skills are considered important because they contribute to general intuitions about numbers and lay the foundation for more advanced skills.
Researchers have linked good number sense with skills observed in students proficient in the following mathematical activities:

- mental calculation .
- computational estimation .


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- judging the relative magnitude of numbers .
- recognising part-whole relationships and place value concepts .
- problem solving .

An intuitive sense of number begins at a very early age. Children as young as two years of age can confidently identify one, two or three objects before they can actually count with understanding. Piaget called this ability to instantaneously recognise the number of objects in a small group 'subitising'. As mental powers develop, usually by about the age of four, groups of four can be recognised without counting. It is thought that the maximum number for subitising, even for most adults, is five. This skill appears to be based on the mind's ability to form stable mental images of patterns and associate them with a number. Therefore, it may be possible to recognise more than five objects if they are arranged in a particular way or practice and memorisation takes place. A simple example of this is six dots arranged in two rows of three, as on dice or playing cards. Because this image is familiar, six can be instantly recognised when presented this way. ${ }^{10}$

Usually, when presented with more than five objects, other mental strategies must be utilised. For example, we might see a group of six objects as two groups of three. Each group of three is instantly recognised, then very quickly (virtually unconsciously) combined to make six. In this strategy no actual counting of objects is involved, but rather a part-part-whole relationship and rapid mental addition is used. That is, there is an understanding that a number (in this case six) can be composed of smaller parts, together with the knowledge that 'three plus three makes six'. This type of mathematical thinking has already begun by the time children begin school and should be nurtured because it lays the foundation for understanding operations and in developing valuable mental calculation strategies.Learning to count with understanding is a crucial number skill, but other skills, such as perceiving subgroups, need to develop alongside counting to provide a firm foundation for number sense. By simply presenting objects (such as stamps on a flashcard) in various arrangements, different mental strategies can be prompted. For example, showing six stamps in a cluster of four and a pair prompts the combination of 'four and two makes six'. If the four is not subitised, it may be seen as 'two and two and two makes six'. This arrangement is obviously a little more complex than two groups of three. So different arrangements will prompt different strategies, and these strategies will vary from person to person. If mental strategies such as these are to be encouraged (and just counting discouraged) then an element of speed is necessary. Seeing the objects for only a few seconds challenges the mind to find strategies other than counting. It is also important to have children reflect on and share their strategies. This is helpful in three ways:

- verbalising a strategy brings the strategy to a conscious level and allows the person to learn about their own thinking;
- it provides other children with the opportunity to pick up new strategies;
- the teacher can assess the type of thinking being used and adjust the type of arrangement, level of difficulty or speed of presentation accordingly. ${ }^{12}$


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